Solutions to Problem 1.

a. When there are n customers in the shop, customers are lost at a rate of $20(n/3)$ customers per hour. Therefore,

$$\begin{aligned}\text{Lost customers / hour} &= 20\left(\frac{0}{3}\right)\pi_0 + 20\left(\frac{1}{3}\right)\pi_1 + 20\left(\frac{2}{3}\right)\pi_2 + 20\left(\frac{3}{3}\right)\pi_3 \\ &= 0\left(\frac{9}{67}\right) + \frac{20}{3}\left(\frac{18}{67}\right) + \frac{40}{3}\left(\frac{24}{67}\right) + 20\left(\frac{16}{67}\right) \\ &\approx 11.34\end{aligned}$$

b. $$\begin{aligned}\text{Expected profit / hour} &= (\text{Expected number of customers / hour})(\text{revenue / customer}) - (\text{cost / hour}) \\ &= \lambda_{\text{eff}}(2) - 4 \\ &\approx (8.6567)(2) - 4 \\ &\approx 13.31\end{aligned}$$

Solutions to Problem 2.

a.

- **State space.** $\mathcal{M} = \{0, 1, 2, \dots, \}$
  Each state represents the number of patients in the urgent care center.
- **Arrival rates.** $\lambda_i = \begin{cases} 2 & \text{for } i = 0, 1, 2, 3 \\ 0 & \text{for } i = 4, 5, \dots \end{cases}$
- **Service rates.** $\mu_i = \begin{cases} 2 & \text{for } i = 1 \\ 4 & \text{for } i = 2, 3, \dots \end{cases}$

b.

$$\left.\begin{aligned} d_0 &= 1 \\ d_1 &= \frac{\lambda_0}{\mu_1} = 1 \\ d_2 &= d_1 \frac{\lambda_1}{\mu_2} = 1\left(\frac{2}{4}\right) = \frac{1}{2} \\ d_3 &= d_2 \frac{\lambda_2}{\mu_3} = \frac{1}{2}\left(\frac{2}{4}\right) = \frac{1}{4} \\ d_4 &= d_3 \frac{\lambda_3}{\mu_4} = \frac{1}{4}\left(\frac{2}{4}\right) = \frac{1}{8} \\ d_5 &= d_4 \frac{\lambda_4}{\mu_5} = 0 \\ \Rightarrow d_j &= 0 \quad \text{for } j = 5, 6, \dots \\ \Rightarrow D &= \sum_{j=0}^{\infty} d_j = \frac{23}{8} \end{aligned}\right\} \Longrightarrow \left\{\begin{aligned} \pi_0 &= \frac{d_0}{D} = \frac{8}{23} \\ \pi_1 &= \frac{d_1}{D} = \frac{8}{23} \\ \pi_2 &= \frac{d_2}{D} = \frac{4}{23} \\ \pi_3 &= \frac{d_3}{D} = \frac{2}{23} \\ \pi_4 &= \frac{d_4}{D} = \frac{1}{23} \\ \pi_j &= \frac{d_j}{D} = 0 \quad \text{for } j = 5, 6, \dots \end{aligned}\right.$$

c. $\ell_q = \sum_{n=s+1}^{\infty} (n-s)\pi_n = (3-2)\pi_3 + (4-2)\pi_4 = \frac{4}{23}$ customers

d. $\lambda_{\text{eff}} = \sum_{i=0}^{\infty} \lambda_i \pi_i = 2\pi_0 + 2\pi_1 + 2\pi_2 + 2\pi_3 + 0\pi_4 = \frac{44}{23}$ customers / hour $\Rightarrow$ $w_q = \frac{\ell_q}{\lambda_{\text{eff}}} = \frac{4}{44} = \frac{1}{11}$ hours

e. Fraction of arriving customers going to Gaussville $= \pi_4 = \frac{1}{23}$